Measurement of Nonuniform Current Density by Magnetic Resonance

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Abstract—Conventional magnetic resonance imagers can measure the electric current density in any substance which can be imaged by nuclear magnetic resonance. This current density image is created by measuring the magnetic field arising from these currents and taking its curl. These magnetic fields are proportional to the phase component of a complex magnetic resonance image. Measurements of all three components of a quasistatic nonuniform current density in a phantom are described. Expected current density calculations from a numerical solution for the magnetic field which was created by the phantom are presented for comparison. The results of a numerical simulation of the experiment, which used this field solution and which included the effects of slice selection and sampling, are also presented. The experimental and simulated results are quantitatively compared. It is concluded that the principle source of systematic error was the finite slice thickness which causes blurring of boundaries. The method could be used to study the spatial distribution of currents injected by an external source if the currents are repetitive and can be synchronized with the magnetic resonance imaging sequence. Physical rotation of the conducting sample is required.

I. INTRODUCTION

Many therapeutic techniques rely on the application of electrical currents to the body, e.g., cardiac defibrillation and pacing, ECT, electrocautery, and numerous treatment methods in physiotherapy. In the field of electrical safety the setting of standards requires knowledge of the currents which result from accidental exposure. Chronic exposure to very small electric and magnetic fields is suspected to have a biological effect. It is therefore relevant to seek methods for the measurement of applied electric currents in tissue. Presently, there is no non-invasive way to do this. A non-invasive current measurement technique and its application to measure a non-uniform current density is described. It could be used to measure applied (i.e., nonbiological) currents. Such currents must be repetitive and synchronized with the magnetic resonance imaging sequence. Physical rotation of the conducting sample is required.

Proposed methods for non-invasive measurement of currents usually involve mapping the magnetic fields outside the current carrying region (for example, with a SQUID magnetometer) [1]–[5] or, alternatively a magneto-acoustic method [6]. Methods of the first type yield a non-unique result unless assumptions such as planar current distributions or dipole sources are made. The method described in this paper requires no assumptions as to the spatial distribution of the current.

Nuclear magnetic resonance imaging can be used to compute the current density \( \vec{J} \) in liquids or gels [7]–[12], [5]. The only requirement is that the medium carrying the current can be imaged. To measure the current density \( \vec{J} \) one first measures the incremental phase \( \Phi \) of a conventional magnetic resonance image with the current flowing. This phase is proportional to the magnetic flux density \( \vec{B} \) which is caused by the current density. Ideally,

\[
\Phi \propto B_z = \vec{B} \cdot \frac{\vec{B}_0}{|B_0|}
\]

(1)

where \( B_z \) is the component of \( \vec{B} \) parallel to the main imaging field \( B_0 \). The current density can then be found using the static version of Maxwell’s equations [13] and the constitutive equations. In the static assumption, the displacement current, \( \partial B/\partial t \), and magnetic induction, \( \partial B/\partial t \), are negligible. Therefore,

\[
\vec{J} = \nabla \times \vec{B}/\mu,
\]

(2)

and the current density \( \vec{J} \) can be determined if \( \vec{B} \) and \( \mu \) are known. Since the materials imaged by magnetic resonance generally have low magnetic susceptibility (\( < 10^{-5} \)), \( \mu \) may be replaced by \( \mu_0 \) in (2).

In a previous paper, the measurement of a single component of a uniform \( \vec{J} \) which was perpendicular to a single axial slice through a cylindrical phantom was reported [9]. Here the measurement of all three components of \( \vec{J} \) in a phantom with a nonuniform current density is reported. Since components of \( \vec{J} \) lying in the plane of the magnetic resonance image were desired, derivatives of \( \vec{B} \) perpendicular to this plane had to be measured. Three different methods for measuring these normal derivatives are compared.
In Section II (Materials and Methods) we describe the phantom, the current imaging pulse sequence, and the postprocessing. Included in the postprocessing are checks on the measured magnetic fields to verify their consistency with the static version of Maxwell's equations. The magnetic field that should have been produced by our apparatus was solved by finite differences. This solution allowed the simulation of the experimental current density formation including the effects of the finite slice thicknesses. The expected current density in the phantom was also computed from the magnetic field solution. The expected current density and the simulated images are compared, in Section III, with the actual images to demonstrate the systematic errors in the measurement and to determine their origin. Consistency checks on the measured magnetic fields are also presented in Section III.

II. MATERIALS AND METHODS

A. The Phantom

The phantom is shown schematically in Fig. 1. It was cylindrically symmetric and consisted of two copper disc electrodes of radius 9.5 mm and thickness 1.6 mm, set at either end of a lucite cylinder, 50.8 mm in height and 50.8 mm in diameter. At the center of the cavity was set an insulating spherical shell (a ping-pong ball) 0.5 mm thick and 37.6 mm in outside diameter. This shell was filled (through a single hole) with CuSO\textsubscript{4} doped saline solution (0.9 g/dl NaCl, 0.1 g/dl CuSO\textsubscript{4}·5H\textsubscript{2}O, \(T_1 = 340\) ms, \(T_2 = 275\) ms). The spherical shell was held in place by hollow post 2 mm in diameter as shown in Fig. 1. The wires which carried the electric current to the electrodes were held in fixed positions with respect to the cell as shown in Fig. 1 and were twisted together as they approached the cell to produce a negligible magnetic source.

B. Imaging Sequence

The magnetic resonance imaging was performed with a General Electric CSI 2 Tesla, 30 cm bore, MR scanner. The current imaging pulse sequence of Fig. 2 was a standard 3-D slab select or 2-D spin echo sequence with the addition of a bipolar current pulse. The readout gradient was 320 Hz/mm and 128 samples of the echo were acquired. The field of view was 65 mm in the slice yielding a pixel separation of 0.52 mm. The repetition period (TR) was 900 ms and the echo time (TE) was 115 ms. The current pulses were 30 mA each with a duration of 50 ms (\(I = 30\) mA, \(T_p = 100\) ms, see Fig. 2).

The current pulse was bipolar so that the phase shifts which it produced were not canceled out by the nonselective 180° pulse. Excitations with a positive/negative current waveform and those with a negative/positive waveform were interleaved to allow compensation for any drift in the imager. Data sets for the + and - current polarities remained separate.

To obtain the components of \(\bar{J}\) which lie in the tomographic plane one must take derivatives of the magnetic field in a direction perpendicular to the slice. Three different slice selection strategies were employed for this purpose. In all three strategies, six slices were obtained with a separation (center to center) of 1.25 mm and offset from the equator from 10.7 to 16.95 mm (see Fig. 1). In the "thin-slice" strategy the slice thickness was also 1.25 mm and was obtained using a 90° pulse with a "sinc" envelope with one sidelobe. In the "overlapping-slice" strategy the slices were 3 mm thick and the envelope had 2 side lobes. In both these strategies adjacent slices were obtained in separate sequences to avoid slice interference. In the "three-dimensional phase encode" strategy, a single slab was selected and subdivided into 8 subspaces by phase encoding in the slice selection direction. The slice
select pulse had two sidelobes and the slab thickness was 7.5 mm. The two outer subslices were positioned outside this slab to avoid aliasing.

To obtain all three components of $\mathbf{J}$ it is necessary to measure all components of the magnetic field $\mathbf{B}$. Because only the component of $\mathbf{B}$ which is parallel to $\mathbf{B}_0$ is measurable, the experiment was repeated for three orthogonal positions of the phantom ($X$, $Y$, and $Z$ in Fig. 3). Positioning accuracy was assured by a jig.

C. Postprocessing

The block diagram of Fig. 4 outlines the post processing steps required to compute the current density $J_z$ perpendicular to an $x$-$y$ slice of the phantom. (To compute $J_x$ and $J_y$, the appropriate cyclic substitutions of $x$, $y$, and $z$, were made.) Four sets of raw data were used, i.e., two orthogonal orientations ($X$ and $Y$ in Fig. 3) each with two current polarities ($\pm$ and $\mp$). Each data set was Fourier transformed to produce $M_{\text{ex}}$—a standard complex MR image, e.g., in the $X$ orientation with a $\pm$ current pulse

$$M_{\text{ex}}(x, y, z) = M(x, y, z) \exp \left[ j \gamma B_z(x, y, z) T_z \right]$$

where $M$ is the continuous transverse magnetization, and $T_z$ is the total current duration (see Figs. 2–4).

For a $\mp$ current pulse and the same ($X$) orientation,

$$M_{\text{ex}}(x, y, z) = M(x, y, z) \exp \left[ -j \gamma B_z(x, y, z) T_z \right].$$

(3b)

The images were rotated and translated to make them coincide with the object's frame of reference (Figs. 1 and 3). A rotation by a multiple of $90^\circ$ in the appropriate direction was used. The centers of mass of the magnitude images were aligned automatically and then a very small ($\pm 1$ pixel) correction was made subjectively. It was necessary, in one case, to stretch the complex image slightly along one axis using bilinear interpolation [14].

The next step was to compute the principal value of the phase (argument) of each pixel and to remove spurious $2\pi$ phase discontinuities by phase unwrapping. Since the curl operation differentiated the phase image, these spurious phase discontinuities caused errors only when they were in the direction of the derivative. Thus, phase unwrapping was only required in the direction of differentiation. When computing derivatives in the slice plane, a "one-dimensional" phase unwrapping algorithm sufficed [15]. When differentiating orthogonal to the slices, phase changes greater than $\pi$ radians could occur due both to the larger slice separation, and to the GE-CSI slice selection process. The "one-dimensional" phase unwrapping algorithm, which assumed that adjacent pixel phase differences exceeding $\pi$ were impossible, would then fail. In this case a "two-dimensional" phase unwrapping algorithm similar to [16] was applied to each plane independently. The relative phases of the planes were then adjusted on the assumption that the interslice phase difference between most corresponding pixels was less than $\pi$ radians. This approach allowed interslice phase differences between some corresponding interior pixels to be
greater than \( \pi \) radians. The images were then scaled by 
\( 1/2 \gamma T_c \) to convert the phase images into magnetic field maps, 
\( \vec{B}_{x\pm}, \vec{B}_{y\pm}, \vec{B}_{z\pm}, \) and \( \vec{B}_{y\mp} \).

Next, the first estimates of the \( z \) components \( \vec{J}_z \) of the current were computed by differentiating the \( \vec{B}_x \) field maps along \( y \) and the \( \vec{B}_y \) field map along \( x \), taking the difference of these derivatives and scaling by \( 1/\mu_0 \). The directional derivatives were computed by convolution with a \( 3 \times 3 \) template. For example, \( \vec{J}_{z\pm} \) was computed as follows:

\[
\vec{J}_{z\pm} = \left[ \frac{\partial \vec{B}_{x\pm}}{\partial x} \frac{\partial \vec{B}_{x\pm}}{\partial y} \right] \cdot \frac{1}{\mu_0} = \frac{1}{8\Delta x} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \ast \vec{B}_{y\pm} \quad - \frac{1}{8\Delta y} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \ast \vec{B}_{x\pm} \cdot \frac{1}{\mu_0}. \tag{4}
\]

These templates arose from the application of Stoke's theorem to a square shaped region centered on the pixel in question and using the trapezoidal rule for integration. The templates form a rectangular contour normal to the desired \( \vec{J} \) component and must be rotated before computing the \( \vec{J}_x \) or \( \vec{J}_y \) components.

\( \vec{J}_x \) is distorted by spatially varying phase errors arising from imperfections in the pulse sequence and instrumentation \cite{17}. By subtracting the \( \vec{J}_{x\pm} \) image from the \( \vec{J}_{x\pm} \) image these systematic errors are removed and the final \( \vec{J}_x \) image is produced. Note that if the subtraction had been performed between the phase calculation and the phase unwrapping steps then, for the unwrapping to function, the maximum allowable phase change in each image would have been \( \pi/2 \) radians per pixel. The former method is more robust since it allowed a maximum phase change of \( \pi \) radians per pixel in each image. When the phase image was computed (as it was for Fig. 6), it would be

\[
\phi(x, y, z) \equiv 2 \cdot \gamma B_j(x, y, z) \cdot T_c. \tag{5}
\]

The number of \( \vec{J}_x \) and \( \vec{J}_y \) images was determined by the effective depth of the derivative template [see (4)]. The template of (4) had a depth of three and produced four images. Templates of depth two would have produced five images in planes midway between the six planes of Fig. 1.

Three methods were used to verify the accuracy of the method. One method, which is described in Section II-D (which follows), compared the experimental current density images against the current density computed for the phantom and against simulated current density images. The other two methods, described below, do not require \textit{a priori} knowledge of the current distribution. They can therefore be used in the usual case, i.e., when the current distribution cannot be computed.

The first of these methods verifies that Maxwell's equation, \( \nabla \cdot \vec{B} = 0 \), is satisfied by \( \vec{B} \) where

\[
\vec{B} = \vec{B}_{z\pm} - \vec{B}_{y\mp}. \tag{6}
\]

The divergence was computed using a \( 3 \times 3 \times 3 \) template. For example, to compute the first partial derivative in the \( z \) direction, we used the template:

\[
\begin{bmatrix}
-1 & -2 & -1 \\
0 & 0 & 0 \\
-1 & -2 & -1 \\
0 & 0 & 0 \\
1 & 2 & 1
\end{bmatrix}
\begin{bmatrix}
1 \\
-2 \\
-4 \\
-2 \\
1 \end{bmatrix}
= \frac{1}{32 \Delta z}.
\tag{7}
\]

This template was derived from the divergence theorem with the surface integral approximated by fractional pixel areas.

The second verification method assumes that the current flows in a region of constant conductivity \( \sigma \) and permeability \( \mu \). Since,

\[
\vec{J} = \sigma \vec{E}, \quad \vec{B} = \mu \vec{H}, \quad \nabla \times \vec{E} = 0, \text{ and } \nabla \cdot \vec{B} = 0,
\tag{8}
\]

and given the vector identity,

\[
\nabla \times \nabla \times \vec{H} = \nabla (\nabla \cdot \vec{H}) - \nabla^2 \vec{H},
\tag{9}
\]

one can show that,

\[
\nabla^2 \vec{H} = \vec{J} \times \frac{\nabla \sigma}{\sigma},
\tag{10}
\]

i.e., in a region of constant conductivity and permeability, the vector Laplacian of \( \vec{H} \) (and \( \vec{B} \)) is zero. The Laplacian was computed by evaluating the second partial derivatives using a \( 3 \times 3 \times 3 \) template and summing them. For example, in the \( z \) direction we used:

\[
\begin{bmatrix} 1 & 2 & 1 \\ -2 & -4 & -2 \\ 1 & 2 & 1 \end{bmatrix}
\begin{bmatrix} -2 \\ -4 \\ -2 \end{bmatrix}
= \frac{1}{16 \Delta z^2}.
\tag{11}
\]

The value of this vector field should be zero at all points inside the phantom and nonzero at points on its insulating boundaries and on the boundaries of the ping-pong ball (except at the poles).

\textbf{D. Magnetic Field and Current Density Calculations}

The magnetic field intensity \( \vec{H} \) was solved for at points inside the phantom. Using \( \vec{H} \), noise-free magnetic reso-
nance images were simulated numerically. The effects of slice shape and thickness, and truncated sampling were included in this simulation. The simulated images were then treated to the same processing as the actual measured data and the resulting simulated current density images compared to the experimental ones. The expected electric current density $\mathbf{J}$ inside the phantom was also computed as

$$\mathbf{J} = \nabla \times \mathbf{H}$$  \hspace{1cm} (12)

from the magnetic field solution $\mathbf{H}$ so that $\mathbf{J}$ could be directly compared to the measured and simulated current density images.

Maxwell's equations for $\mathbf{H}$ were solved by a combination of an analytic and a numerical technique. The symmetric contribution to $\mathbf{H}$ arising from the lead wires was solved analytically. The contribution to the field $\mathbf{H}$ arising from volume currents was solved using the method of finite differences [18]. The difference equations were set up in cylindrical coordinates based on the integral form

$$\oint_c \nabla \times \mathbf{H} \cdot d\mathbf{l} = 0$$  \hspace{1cm} (13)

of

$$\nabla \times \nabla \times \mathbf{H} = 0,$$  \hspace{1cm} (14)

which is a consequence of the conductivity in the phantom being constant. The cylindrical symmetry of the phantom implied that only $H_\phi$, the tangential component of $\mathbf{H}$, existed and was independent of $\varphi$. Furthermore, since the ball was centered between the identical electrodes, only one quadrant needed to be computed. The integral was approximated by a five point formula computed over a square contour in the $r$-$z$ plane. These square cells were arranged in a $99 \times 99$ grid. The resulting cells were 0.257 mm to the side, half the size of the pixels in the experiment, and approximately 5 cells corresponded to the slice (subslice) separation of 1.25 mm. Each node in the grid generated a difference equation. $H_\phi$ was zero on the $z$ axis and on the ball surface. It equalled $I/2\pi r$ on the outer insulating boundaries. At the electrode surface and on the $z = 0$ plane, $\partial H_\phi / \partial z = 0$.

The computation of the simulated current density images accounted for the effects of slice thickness by numerical integrations in real (as opposed to "k") space. First the slice "profiles" were computed from a numerical integration of the Bloch equations. The appropriate truncated "sine" RF excitation waveform, tuned for a maximum flip angle of 90° [19], was used. A constant spin density was assumed except within the spherical shell of the ping-pong ball. The magnitude of these "slice (slab) profiles" $M$ are plotted in Fig. 5. To account for sub slice encoding in the "three-dimensional phase encode" case, the slab profile was convolved with the function:

$$e^{-j\pi z/8\Delta z} \cdot \frac{\sin (\pi z/\Delta z)}{\sin (\pi z/8\Delta z)},$$  \hspace{1cm} (15)

Phase variation across the profile was corrected as much as possible by a linear phase shift normal to the slice. The resulting profile was inserted for $M$ in the right side of (3). The value of $B_j$ in the right side of (3) was determined directly from the finite difference solution just described and the sign of the argument from the current po-
Fig. 6. (a) Magnitude, (b) phase, and (c) magnetic flux density images for the slice at an offset of 14.45 mm from the equator of the ping-pong ball. The range of values shown in (b) is ±π radians and in (c) it is ±400 nT. The "thin-slice" slice selection strategy was used.

III. RESULTS

Fig. 6 shows the magnitude (a) and phase (b) of the MRI image of the slice offset at 14.45 mm (from the equator) using the "thin slice" slice selection strategy. Although the current was flowing when these images were made, the resulting fields were not strong enough to significantly affect the magnitude images. The shell of the ping-pong ball appears as a dark ring concentric with the circular cross section of the phantom. The four hollow supporting posts appear as small circles at the "compass
points.' The seam of the ball is also visible. The darker circle inside the ball was attributed to imperfections in the imager and is representative of the artifacts seen in the magnitude images. The phase image Fig. 6(b) was made by subtracting the phase images formed with opposite current polarities as in (5). Thus, the phase variations in Fig. 6(b) are entirely due to the current. Pixels with a low signal-to-noise ratio have been masked out, e.g., at the ball boundary. The artifacts visible in the magnitude image Fig. 6(a) do not appear in the phase image Fig. 6(b). The spurious 2\pi phase changes seen in Fig. 6(b) have been removed in Fig. 6(c), by "two-dimensional" phase unwrapping, and the image scaled to yield the magnetic field. Fields in the range \pm 400 nT were measured near the return wire. Note that the magnetic field is nonzero inside the shell of the ping-pong ball.

Fig. 7 shows the current images computed for the same slice selection strategy as in Fig. 6 but for all four central
slices. In Fig. 7(a) and (b), the $J_x$ and $J_y$ components are of the opposite sign to their respective $x$ or $y$ coordinates. These figures display the magnitudes of the current densities. The scale on all images in Fig. 7 is 0 to 24 $\mu$A/mm$^2$. The major observation to be made from Fig. 7 is that no current appears inside the ping-pong ball. Note also that, as the slices approach the center of the ping-pong ball, the region of zero current increases in diameter and the $J_x$ and $J_y$ decrease in magnitude. Finally, the circular artifact in the current carrying region that is just visible in some of the slices in Fig. 7 and which can be seen in the original of Fig. 6(a) is attributable to machine imperfection. The artifact seen inside the ball in Fig. 6(a) does not appear in Fig. 7.

Fig. 8 shows the currents calculated using the "three-dimensional phase encode" (b) and "overlapping slice"
(a) strategies. These should be compared to the top right images in Fig. 7 as they are for the same slice. Again, we see no current or artifact inside the ball.

In Fig. 8(c), we present, for the slice at 14.45 mm, the divergence magnitude (lower right) and the vector Laplacian component magnitudes of the measured magnetic fields. The divergence should be identically zero and the Laplacian should be zero in the uniformly conducting regions of the phantom. The divergence has the same dimensions as current density (\(\mu A/mm^2\)) so it is possible to compare it quantitatively with the current density images of Figs. 7 and 8. The current density range in Figs. 7 and 8 is 0 to 24 \(\mu A/mm^2\) while that in Fig. 8(c) is only 0 to 2.4 \(\mu A/mm^2\). When the divergence image is compared to the magnitude images of Fig. 6(a), regions of large divergence correspond to regions showing imaging artifacts or to the ball boundary. The dimensions of the vector Laplacian (\(\mu A/mm^3\)) shown in Fig. 8(c) are not comparable to any of the other data. The pattern and polarity of the values at the ball boundary is qualitatively predicted by (10) [see Section IV, Discussion].

As a further check on the accuracy of the method, the net current flowing in the \(z\) direction was calculated by summing the pixel values in each current image. It should equal the 30 mA current that was applied. The largest error was a 1.5 mA (5%) current loss for the "overlapping slice" technique. This is within expected experimental accuracy of the nominal current.

Fig. 9 shows graphs of current density versus radial distance for points along a diagonal line from the upper left to the lower right-hand corner of the current images. The values shown are bi-linear interpolations of the current densities directed along this line (in the noted direction) \(J_y\) Fig. 9(b) and along the \(z\) axis \(J_z\) Fig. 9(a). The experimentally measured values are represented as points with error bars whose length is twice the standard deviation of 121 pixels at the center of the experimental current density image. The spatial separation of these points is 0.735 mm (i.e., \(\sqrt{2} \cdot Ax\)). Values from simulated current images are displayed as solid lines and the expected current densities [computed using (12)] as dashed lines. In these figures, each of the three columns is associated with
a different slice selection strategy and each row with a different slice offset from the equator of the ping-pong ball.

For all the cases, the simulated image current density and the expected current density are identical except at the ball boundary. At this boundary the expected current density shows a monotonic transition from zero to peak over a distance of about 2.5 pixels.

As with Figs. 7 and 8 the current inside the ball is essentially zero in all cases. The agreement between the experimental and simulated current density images is generally very good, particularly for the radial $J_r$ densities. With the exception of pixels in the transition region near the ball boundary, the largest difference is 2 $\mu$A/mm$^2$.

In the "thin slice" $J_r$ upper image, isolated differences between experiment and simulation appear (about 7 mm out from the ball). These errors are associated with a large circular artifact, barely noticeable in Fig. 6(a). At the ball boundary both the experimental and simulated "thin slice" images of the uppermost slice show a monotonic transition from zero to peak current over a distance of 6.2 mm for $J_r$ and 4.8 mm for $J_z$. This transition distance drops to 2.8 mm and 2.0 mm for the lowest slice. Disagreement between simulated and experimental values is about 4 $\mu$A/mm$^2$ in the transition region.

For the "three-dimensional phase encode" technique, the simulations predict a transition region of width 9 (top) to 6.5 mm (bottom) for the $J_r$ images and that the current density will oscillate as the region is crossed. The experimental points follow a similar pattern but some pixels can differ by 12 $\mu$A/mm$^2$ from the simulation in the transition region. In the $J_z$ "three-dimensional phase encode" images the transition region varies from 9 (top) to 4 mm (bottom). The transition is monotonic for both simulated and experimental points and individual pixels can differ by 10 $\mu$A/mm$^2$.

For the "overlapping" slice method the simulations predict a transition region of width 8 (top) to 5.5 mm (bottom) for the $J_r$ images and that the current density will oscillate as the region is crossed. The experimental points follow a similar pattern but some pixels can differ by 8 $\mu$A/mm$^2$ from the simulation in the transition region. In the $J_z$ "three-dimension phase encode" images the transition width is from 9 (top) to 6 mm (bottom). The transition is monotonic for both simulated and experimental points and individual pixels can differ by 4 $\mu$A/mm$^2$. 
Fig. 10. The measured current densities for all slice selection strategies plotted vectorially. The current magnitude should increase monotonically toward the ball boundary.

Such a phase shift was observed in the upper slices when imaging in the z direction with earlier sequences in which slice dephasing and rephasing gradients were not contiguous. This phase artifact was unambiguously identified with MHD flow since the phase reversed sign when the current polarity was reversed. The artifact disappeared when the phantom was filled with a gel (0.5 g/ml agarose) [24] or by imaging with the sequence of Fig. 2, in which concatenated gradients reduced motion sensitivity. Furthermore, the Lorentz force vorticity, \( \nabla \times (J \times \hat{B}_0) \), was nonzero in the region of current flow—indicating a force that could spin the fluid. The Lorentz force flow artifact was so small (0.1–1 mm/s) that gradient moment nulling or inherently refocused RF pulses [25] were unnecessary.

Magnetic resonance imaging artifacts may produce artifacts in corresponding positions in the current density images. The subtraction of \( J_T \) from \( J_\perp \) (see Fig. 4) will remove artifacts which do not change between current polarity reversals. Thus, normal imaging artifacts are no more troublesome with current density images than with standard magnetic resonance images.

The qualitative effect of the slice selection strategy on current density images and standard MR images is similar. In the “three-dimensional phase encoded” \( J_z \) image Fig. 8(b) the bright annular “rind” surrounding the ball is a direct result of interslice ringing introduced by the Fourier encoding point spread function seen as a dotted line in Fig. 5(b). In contrast, the more rectangular shape and smaller and fewer side lobes of the “overlapping” slice profiles produce less “leakage.” For these slices, the transition region comprises pixels which are traversed by the ball boundary within the nominal slice width. In these pixels, the phase is weighted towards points with higher spin density.

Since spatial structure of the \( \hat{B} \) images is low, they must be differentiated to reveal their beauty. The \( 3 \times 3 \) template used for this purpose was superior to a \( 2 \times 2 \) template that was tested because it suppressed ringing due to truncation of the FID. Any form of numerical differentiation will also reduce spatial resolution. This effect can be seen most clearly in the expected current graphs (Fig. 9) which have a transition region of 2.5 pixels arising from the differentiation required for their computation [see (12)].

The use of overlapping slices will result in a higher signal but also in longer imaging times than the thinner non-overlapping slices. This SNR advantage will still be realized with equal imaging times when more averages can be taken of the nonoverlapping slices. The disadvantage of thicker slices is loss of resolution and potential loss of signal due to intra voxel phase cancellation. The three-dimensional phase encoding technique has little to recommend it but simplicity [26].

Inter-pixel phase changes must be less than \( \pi \) for most pixels. Higher values are acceptable for a few pixels between planes provided that phase unwrapping is done with
care. Higher values in the imaging plane will lead to signal loss. It is paradoxical that, for given voxel size, as one increases the current (i.e., the signal) the current signal-to-noise will reach a maximum and then rapidly decrease. The cure for this loss of SNR is to increase the spatial resolution [12].

The divergence test for artifacts will be very useful when measuring all three components of \( \mathbf{J} \). The required derivatives of \( \mathbf{B} \) in the direction of \( \mathbf{B}_0 \) are easily acquired. The test will reveal all types of artifacts. The vector Laplacian test can also be used but its interpretation requires the assumption of constant conductivity, i.e., a priori knowledge of the object. As seen from (10), the Laplacian is nonzero only when \( \sigma \) changes and \( \mathbf{J} \) is neither zero nor parallel to \( \nabla \sigma \). In samples of unknown conductivity structure, the vector Laplacian will detect conductivity boundaries with \( \mathbf{J} \) controlling the contrast.

If only one component of \( \mathbf{J} \) is being measured, the divergence test cannot be used. However, two components of the vector Laplacian can be computed if \( \mathbf{B} \) is measured in adjacent slices. For example, if only \( J_z \) is sought then \( \mathbf{B}_x \) and \( \mathbf{B}_y \) must be measured. It is, therefore, easy to compute

\[
\nabla^2 \mathbf{B}_x = \frac{\mu_0}{\sigma} (\mathbf{J} \times \nabla \sigma)_x,
\]

and

\[
\nabla^2 \mathbf{B}_y = \frac{\mu_0}{\sigma} (\mathbf{J} \times \nabla \sigma)_y,
\]

with the \( z \) derivatives, which are not required to compute \( \mathbf{J} \), coming from the adjacent slices. These images can be used as a test for artifacts if \( \nabla \sigma \) is known to be zero. They also show boundaries between regions of differing conductivity.

When this technique is applied to living animals, several sources of artifact not encountered in this study will arise. First, the applied currents will cause muscles to twitch. This synchronous motion can cause localized phase changes [27] which would produce artifacts in the current images. Second, there is an upper limit to the current that can be applied without ill effect. This limit will determine how much current can be tolerated and therefore the spatial resolution. Measurements have been reported [11] in dead tissue in which 24 mA applied current yielded a peak current density of \( 9 \pm 0.3 \) mA/mm\(^2\) with a 1.5\(^2\) \times 5 mm voxel. It seems unlikely that spatial resolutions less than 3 mm [12] will be easy to achieve in vivo. Finally, at least one rotation of 90\(^\circ\) about an axis perpendicular to the main imaging field is required. Even when this axis is vertical, some deformation of tissue may occur and produce artifact.

V. CONCLUSION

We conclude that it is possible to measure all components of a nonuniform current density accurately using this magnetic resonance imaging technique. The current must be repetitious and synchronized to the imaging sequence and the sample must be rotatable into at least two orthogonal positions. Artifacts can be detected by fundamental tests.

The major determinant of systematic error is the sampling interval and slice thickness chosen for imaging. Tradeoffs between SNR, imaging time, voxel volume, and resolution are made on the same basis as with conventional MR imaging with one exception. The differentiation required in the computation of current density makes the current density SNR more sensitive to spatial resolution than the conventional magnitude MR image.

Because magnetic resonance can image biological tissue and current density, it is potentially suitable for the study of therapeutic current flow in biological tissue.

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