A Local Formula for Inhomogeneous Complex Conductivity as a Function of the RF Magnetic Field

Adrian Nachman[1,3], Dinghui Wang[2,3], Weijing Ma[2,3], Michael Joy[2,3]
University Of Toronto [1]Mathematics,[2]Institute Biomedical & Biomaterial Engineering and [3]Electrical & Computer Engineering

Summary: If only we had measurements of the full RF magnetic field inside the body, we could calculate the corresponding inhomogeneous complex conductivity by a new explicit formula. This formula requires only the values of the magnetic field near the points of interest. Global knowledge of the fields is not needed. We present reconstructions of electric permittivity and inhomogeneous conductivity using this formula. These are based on numerically computed RF magnetic field in material models.

Introduction: It is relatively easy to use MRI to measure a non uniform transmit field relative to a uniform receive field at frequencies for which there is little interaction with the body. The RFCDI method[1] is based on such measurements. At higher frequencies in which such interactions are strong, the receive field is no longer uniform. We know of no method that measures the absolute magnetic field under these conditions.[1,2,3] This is the unsolved problem.

Recently there has been significant interest in using the troublesome interaction between the transmit field and the human body at high fields to measure the complex electrical conductivity of the body without additional applied currents[2,3]. We present here a way to directly compute $\sigma + j\omega\varepsilon$ from knowledge of the magnetic field if/when its measurement becomes practical. With such a measurement one can also, of course, obtain current density.

The Formula: Starting with Maxwell's equations and letting $\sigma + j\omega\varepsilon$ denote the inhomogeneous isotropic complex conductivity,

$$\nabla \times E = -j\omega\mu_0 H,\ \nabla \cdot H = 0 \text{ and } \nabla \times H = (\sigma + j\omega\varepsilon) E$$

Taking the curl of the third equation and substituting the second

$$\nabla \times (\nabla \times H) = \frac{\nabla(\sigma + j\omega\varepsilon)}{(\sigma + j\omega\varepsilon)} \times (\nabla \times H) - j\omega\mu_0 (\sigma + j\omega\varepsilon) H$$

Taking the scalar product with $(\nabla \times H)$ and using $A \cdot (B \times A) = 0$

$$[\nabla \times (\nabla \times H)] \cdot (\nabla \times H) = -j\omega\mu_0 \gamma H \cdot (\nabla \times H)$$

Using $\nabla \cdot H = 0$ and $\nabla \times (\nabla \times H) = \nabla(\nabla \cdot H) - \nabla^2 H$ we get our formula for the complex conductivity

$$\sigma + j\omega\varepsilon = -j \frac{(\nabla^2 H) \cdot (\nabla \times H)}{\omega\mu_0 H \cdot (\nabla \times H)}.$$

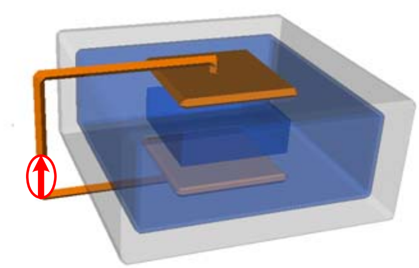

Fig.1 A model for simulation

This formula can be used at all points where the electric and magnetic field are not perpendicular.

Numerical tests: Models with various electrical properties and electrode configurations have been tested. One model is shown in Fig. 1. A rectangular plastic container is filled with conductive materials. Through electrodes and wires, sinusoidal RF current at 64MHz is applied. The electromagnetic fields were computed by 3-D FDTD. Steady-state responses in time domain were converted to phasor form. The results of one transverse plane are shown in Fig.2. The conductivity of the center part varies linearly along x direction. In Fig. 2 (b), the reconstructed conductivity in the center part is averaged along y direction and compared to the original one.

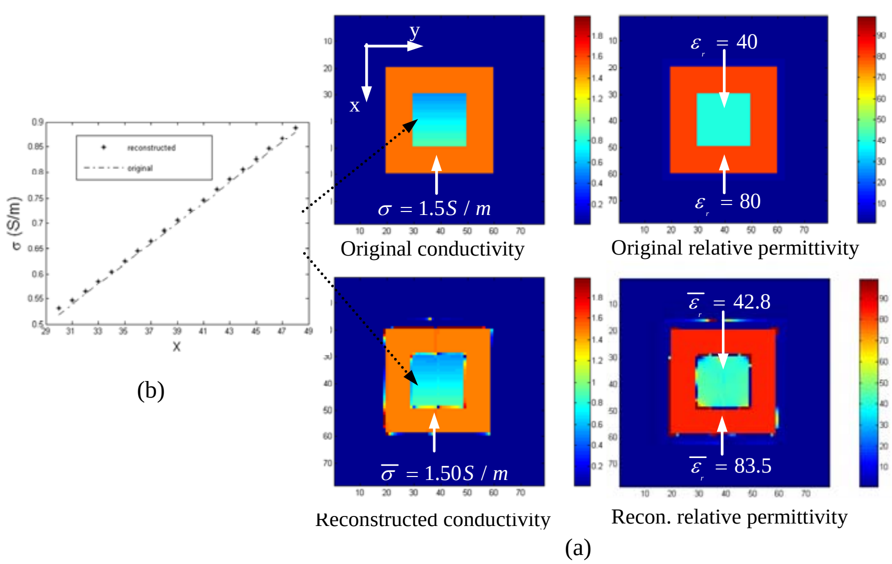

Fig. 2 Results of numerical reconstruction corresponding to the configuration shown in Fig.1

Conclusion: Simulations show that even at 1.5T for realistic material properties (but no added noise) the angle between the electric and magnetic field allows accurate reconstruction of the inhomogeneous conductivity by our formula.

References:
[1] G. C. Scott, et al. Electromagnetic considerations for RF currentcensity imaging. IEEE Trans. Med. Imag., Vol. 14, No.3, pp. 515-524, 1995.
[2] F. Wiesinger, et al. Prospects of absolute B_1 calibration. Unsolved Problems and Unmet Needs in Magnetic Resonance, ISMRM 14, 2006.
[3] U. Katscher, et al. Electric properties tomography (EPT) via MRI. Proceedings of ISMRM 14, pp. 3037, 2006.